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Principles of a new generation of simplified and accurate watt balances

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Abstract

It has been known for some time that the watt balance, which is based on the virtual work principle, is insensitive to some misalignments which would, at first sight, be expected to produce significant errors. In this paper we show that, in particular circumstances, this insensitivity applies to a whole range of misalignments. This effect can be exploited to design a watt balance, with a constrained coil motion, which offers advantages over conventional designs. We present three such new designs: one is based on a conventional balance and the other two are based on strip hinges, which are known to have excellent properties when measuring small force differences and with the production of precise linear movements having little motion in the other five degrees of freedom. A constrained design of this kind would have advantages for reproducing the forthcoming SI definition of mass in the range from milligrams to kilograms, whenever and wherever desired, and might well do so with improved accuracy.

Keywords: watt balance, redefinition, kilogram, Planck constant, mass measurement, gravimetry

(Some figures may appear in colour only in the online journal)

1. Introduction

In general, the coil of a watt balance [1] will have six degrees of freedom giving rise to three orthogonal components (u_x, u_y, u_z) of its linear velocity $\boldsymbol{u}$ and three orthogonal components $(\omega_x, \omega_y, \omega_z)$ of its angular velocity $\boldsymbol{\Omega}$. The coil may also generate a force $\boldsymbol{F}$ having components (F_x, F_y, F_z) and a torque $\boldsymbol{\Gamma}$ having components $(\Gamma_x, \Gamma_y, \Gamma_z)$ [2]. These quantities may be combined to yield the general expression governing the watt balance, as follows.

If a current I flows in a coil linked by a magnetic flux Φ the external force $\boldsymbol{F}$ and torque $\boldsymbol{\Gamma}$ acting on the coil are given by:

$$\boldsymbol{F} = -I\,(\boldsymbol{i}\partial\Phi/\partial x + \boldsymbol{j}\partial\Phi/\partial y + \boldsymbol{k}\partial\Phi/\partial z) \qquad (1)$$

and

$$\boldsymbol{\Gamma} = -I\left(\boldsymbol{i}\partial\Phi/\partial\theta_x + \boldsymbol{j}\partial\Phi/\partial\theta_y + \boldsymbol{k}\partial\Phi/\partial\theta_z\right), \qquad (2)$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are orthogonal unit vectors along the x, y and z directions respectively.

If the current is switched off and the coil moved with velocity $\boldsymbol{u} = (\boldsymbol{i}u_x + \boldsymbol{j}u_y + \boldsymbol{k}u_z)$ and angular velocity $\boldsymbol{\Omega} = \left(\boldsymbol{i}\omega_x + \boldsymbol{j}\omega_y + \boldsymbol{k}\omega_z\right)$ a voltage V is induced in the coil, where

$$V = -\big(u_x\partial\Phi/\partial x + u_y\partial\Phi/\partial y + u_z\partial\Phi/\partial z + \omega_x\partial\Phi/\partial\theta_x + \omega_y\partial\Phi/\partial\theta_y + \omega_z\partial\Phi/\partial\theta_z\big). \qquad (3)$$

If the coil and flux are identical for both measurements, the rates of change of flux in space in (1), (2) and (3) can be eliminated to yield the fundamental exact equation:

$$VI = \boldsymbol{F}.\boldsymbol{u} + \boldsymbol{\Gamma}.\boldsymbol{\Omega} \qquad (4)$$

or

$$VI = \left(F_x u_x + F_y u_y + F_z u_z + \Gamma_x\omega_x + \Gamma_y\omega_y + \Gamma_z\omega_z\right). \qquad (5)$$

Equation (5) will yield an accurate result for the measurement if all the components on its right-hand side are accounted for. If it cannot be shown that these components are correctly accounted for it is usual to treat each such component as a source of uncertainty. An alignment procedure is then carried out to minimize each such component and ensure that the combination of these uncertainties does not contribute significantly to the target uncertainty which, at present, is 2 parts in 10^8. This requirement increases the complexity of

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the apparatus, increases the difficulty of its operation and, to date, has provided a significant obstacle to obtaining the target uncertainty. The horizontal force terms and the torque terms reflect the quality of the alignment between the coil and the magnetic flux; a perfectly aligned coil would produce only a vertical force with no moment about its centre of mass.

2. General conditions for insensitivity to torques and horizontal forces

Previous work [3] showed that the result from a watt balance, which uses a beam balance for both moving and weighing modes of the experiment, was insensitive to weighing with the beam offset from the horizontal, which would be expected to give an increased error term $F_x u_x$ arising from the finite velocity u_x at the weighing point and the sensitivity of the offset balance to horizontal forces F_x. This somewhat counter-intuitive result is the result of a precise cancellation of the errors arising in the weighing and moving phases of the experiment. This result can be generalized to show that for a particular class of watt balances, having a common moving and weighing mechanism, there are no errors resulting from similar misalignments.

If the apparatus coil assembly is designed to be rigid and is attached to the balance so that its motion along and about all its axes is determined by the vertical (z'-axis) velocity $u_{z'}$ of the point (z') on the balance where the weight of the mass is applied, then we can write:

$$\begin{aligned} u_x &= u_{z'}\partial x/\partial z', \quad & u_y &= u_{z'}\partial y/\partial z', \quad & u_z &= u_{z'}\partial z/\partial z' \\ \omega_x &= u_{z'}\partial \theta_x/\partial z', \quad & \omega_y &= u_{z'}\partial \theta_y/\partial z', \quad & \omega_z &= u_{z'}\partial \theta_z/\partial z', \end{aligned} \tag{6}$$

where the partial derivatives will either be constants or functions of z'. Equation (3) then becomes:

$$\begin{aligned} V = -u_{z'}(&\partial\Phi/\partial x \cdot \partial x/\partial z' + \partial\Phi/\partial y \cdot \partial y/\partial z' \\ &+ \partial\Phi/\partial z \cdot \partial z/\partial z' + \partial\Phi/\partial\theta_x \cdot \partial\theta_x/\partial z' \\ &+ \partial\Phi/\partial\theta_y \cdot \partial\theta_y/\partial z' + \partial\Phi/\partial\theta_z \cdot \partial\theta_z/\partial z'). \end{aligned} \tag{7}$$

In weighing mode the forces arising from the coil will be in static equilibrium with the weight Mg of the mass acting in the direction $-\boldsymbol{k}$. We are interested in the resultant of the forces and torques acting along the z'-axis so, by the method of virtual work, we can write for the forces acting along the vertical (z'-axis)

$$\begin{aligned} 0 = -Mg - I(&\partial\Phi/\partial x \cdot \partial x/\partial z' + \partial\Phi/\partial y \cdot \partial y/\partial z' \\ &+ \partial\Phi/\partial z \cdot \partial z/\partial z' + \partial\Phi/\partial\theta_x \cdot \partial\theta_x/\partial z' \\ &+ \partial\Phi/\partial\theta_y \cdot \partial\theta_y/\partial z' + \partial\Phi/\partial\theta_z \cdot \partial\theta_z/\partial z'). \end{aligned} \tag{8}$$

If the partial derivatives of the spatial coordinates are unchanged between moving and weighing modes equations (7) and (8) can be combined to eliminate terms in Φ, giving:

$$Mg = IV/u_{z'}$$

or

$$Mgu_{z'} = IV. \tag{9}$$

Equation (9) resembles the usual expression for the equality of virtual mechanical and electrical power in a watt balance, except that it is now independent of horizontal forces and torques, provided that the virtual displacements implicit in (6) and (8) do not vary between moving and weighing modes, and provided that there are no significant forces or torques, other than those produced by the interaction of current and magnetic flux. This favours existing watt balance designs which use the same mechanism for both moving and weighing, such as the NPL and NIST watt balances, where the relatively unrestrained coil can be aligned well enough to ensure that the above requirements are met. In the watt balances described in this paper these requirements are met due to their construction, but, as in all precise metrology, additional careful alignment of any such apparatus would be a prudent precaution. From the derivation in [3], equation (9) also applies to a composite coil if, for each component coil, the virtual displacements and the coil resistance remain constant between moving and weighing modes.

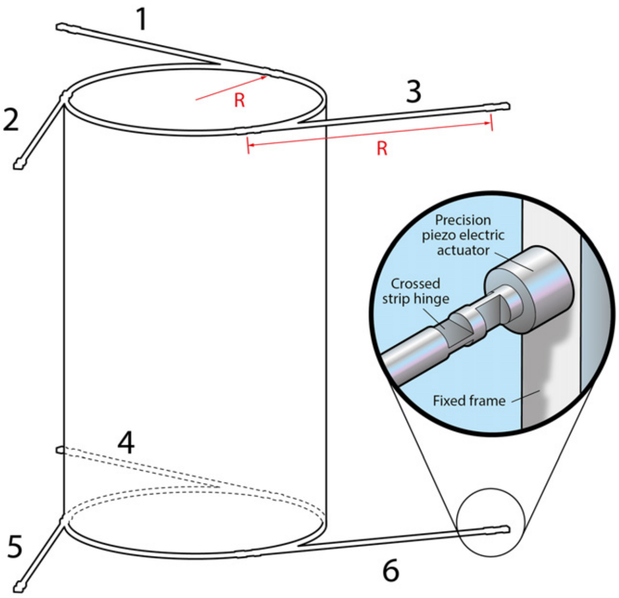

Figure 1. The 'seismometer' suspension showing details of the flexures at either end of the rods and the piezo actuators to control axial rotation. The precise dimensions of the strip hinges would be set to provide the best sensitivity during weighing whilst ensuring that the apparatus remained robust.

In the original NPL and NIST watt balance designs the coil is suspended as a pendulum and therefore the suspension provides only a weak resistance to displacement and motion in the five unwanted degrees of freedom. Here we propose alternative suspension mechanisms offering more than a thousand times greater elastic resistance. Consequently the horizontal velocities u_x and u_y, the angular velocities ω_x, ω_y and ω_z and the terms in (5) containing them may be reduced, and are likely to remain unchanged over a substantial time interval. We suggest an alignment procedure that will yield a similar result to those procedures presently employed to minimize the forces and torques which can affect the critical assumptions made above and thereby provide an accurate measurement of equivalent mechanical and electrical virtual power.

3. ‘Seismometer’ suspension

Figure 1 shows a cylindrical object mounted using a ‘seismometer’ suspension which consists of two sets of three equal-length rods 120° apart in direction lying in two widely separated parallel planes perpendicular to the axis of the object (z-axis). The rods are freely jointed to the object at one end and to fixed anchoring points at the other by crossed strip hinges and meet the suspended object tangentially.

The remarkable property of this suspension is that it allows completely free movement of the object for a short distance along its axis whilst providing a stiff constraint to motion in the remaining two orthogonal directions and the three angular degrees of freedom. In principle, only five arms are required, but the overdetermined addition of a sixth arm does not alter these characteristics significantly if reasonable manufacturing tolerances are achieved. The sixth arm ensures much greater stiffness for some of the unwanted degrees of freedom. Free linear axial translation is accompanied by a small axial rotation proportional to z^2 as the suspended object moves a distance z along its axis away from its symmetrical position where the arms are coplanar. This rotation is minimized for a given overall size of the apparatus if the length R of each suspension arm is equal to the distance R from the axis to the point at which the arm is attached to the suspended object. The suspended object is not necessarily a thin-walled cylinder. A lattice framework providing anchoring points for the suspension arms would be lighter and more practicable.

4. Application to a watt balance

Figure 2 illustrates the basic design of a watt balance using the structure shown in figure 1. The whole apparatus is contained within an evacuated vessel to remove the buoyancy effect of air on the mass M and the effect of the refractive index of air on interferometric length, velocity and acceleration measurements.

The change in the force generated by a current in the measuring coil F positioned in the radial magnetic flux of the permanent magnet G, shown at the bottom of the diagram, opposes the vertical force of Mg newtons produced by a mass M (usually 1 kg) in the Earth’s gravitational acceleration, g. There is no conventional balance arm and the strip hinges are the equivalent of the knife-edges and planes; strip hinges are known to work very well in a conventional balance [4]. The role of a tare mass on the other arm of a conventional beam balance is here provided by a second current-carrying coil in a second inverted magnet A at the top of the diagram. By adjusting the current in this ‘tare’ coil the total mass of the suspended system (without the 1 kg mass to be calibrated) can be opposed and manipulated slightly for execution of the moving phase. In the weighing phase the force of the total suspended mass (without M in position) plus an upwards force of $Mg/2$ newtons can be applied so that, to attain equilibrium, a current of $-I/2$ in the measuring coil is required. When the mass M is positioned on the scale-pan, using mass lift B, a current of $+I/2$ restores equilibrium by supplying an upwards force of $Mg/2$ newtons. The total change of I in the current then corresponds to the total force of Mg newtons exerted by M. This commonly employed strategy ensures that the effects of the currents $+I/2$ and $-I/2$ on the magnetic material of the measuring magnet are symmetrical and the resulting linear changes in the magnetic field cancel. This symmetry can be further reinforced by a symmetrical design of measuring coils and magnet, as used in the NPL Mk II watt balance [5]. Care must be taken with the design of the upper coil and magnet as the coil needs to be supplied with a significant current to oppose the weight of the moving part of the apparatus. The heating effect of this current must not be allowed to affect the temperature of the magnet and its magnetic field as this would compromise the stability of the tare force in the weighing mode. Ideally this force should be constant, but a small, predictable, drift is acceptable.

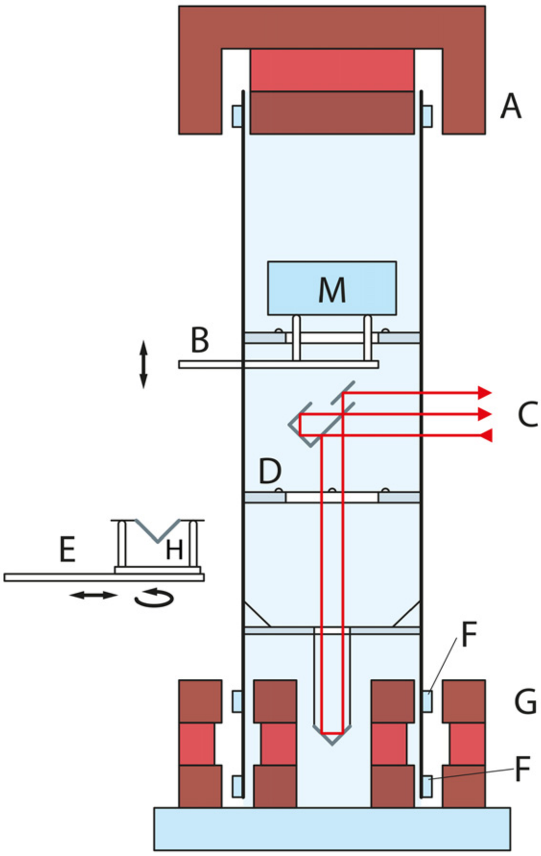

Figure 2. A cross-section of a possible watt balance based on the ‘seismometer’ suspension. The tare magnet and coil is labelled A. B is the lift mechanism for mass M. C is the interferometer for measuring the position, velocity and acceleration of the frame that supports the coils and the mass. D, E and H are components used to configure the apparatus as a gravimeter. The main coils F are mounted on the frame and are in the radial field of magnet G.

The voltages generated by the apparatus can be measured using the Josephson effect. The current can be measured from the voltage drop it produces across a resistor calibrated in terms of quantum Hall resistance and the velocity can be measured interferometrically. These measurements would be identical to those encountered in other watt balances.

The interferometer cube-corner is rigidly fixed to the framework which supports the mass in weighing mode and thus measures the required velocity of the point which is where the weight of the mass is applied to the balance in weighing mode. The assembly of the beam-splitter and second cube-corner, which completes the interferometer at C, is rigidly attached to the magnet via the supporting framework of the apparatus.

This design of this watt balance makes possible a simple modification to allow it to operate as an absolute gravimeter based on interferometric observation of the free-fall of the cube-corner retroreflector, H, which can be kinematically mounted on plate D to intercept the light in the moving arm of interferometer C. This requires a second positional monitoring system (not shown) to control the position of the frame during loading of the retroreflector and subsequent operation as a gravimeter. The retroreflector is moved in and out of its position on the frame by the mechanism E which moves H onto the axis of the apparatus whereupon the framework descends and H passes through slots in D. The mechanism E then rotates the retroreflector by 60° so that it can engage with the kinematic mounting on D when the apparatus frame is raised and E is retracted.

The interferometer can measure the acceleration of the retroreflector H when it is either propelled upwards and released by a rapid motion of the suspended framework, or simply released to fall freely. In either case, M is lifted out of the way so that it does not contact the rapidly moving framework. The motion of the framework would be under the control of the current in the upper (drive) coil so that it moves along with the retroreflector to provide a guard around it to protect it from the viscous drag of the residual gas in the evacuated vessel. Care must be taken to isolate the fixed part of the interferometer both from ground vibration and from the vibrations arising from the forces used to control the motion of the suspended framework; this must be achieved in addition to ensuring that the interferometer also measures velocity correctly in the moving mode of the watt balance. This gravimeter could measure g at times interspersed with the moving and weighing phases, thus avoiding the expense of a separate absolute gravimeter and the errors associated with time interpolation of tidal and atmospheric changes of g. The problems of spatial interpolation of g values to the mass centre of M are also minimized, since there would be no horizontal corrections and only a difference in g of a few parts in 10^8 between the positions of H and M. The observation distance available from a single drop would be limited to a few tens of millimetres by the maximum displacement of the suspended framework. This is less than that available to existing commercial gravimeters [6], but is comparable to some other absolute instruments such as the cam-operated gravimeter described in [7]. If the mechanism can be reset rapidly the effect of this restricted observation distance/time on the resolution of the instrument can be partially compensated by the increase in the number of drops which can be performed in a given time. This should allow the instrument to achieve a resolution sufficient to support the operation of the watt balance with an operating time in the usual 30 min to 60 min range.

An apparent disadvantage of this strip hinge mechanism for a watt balance is that linear axial translation is accompanied by the small axial rotation. But if the velocity/voltage measurement is fitted to its value at the point where the force/current measurement is carried out there is no added error. Additionally, if this point is the symmetrical position (as is usual with present apparatuses) any residual effect is small because the rotational angles and velocities are zero at this point. To reduce possible errors in the interferometry caused by the rotation it is possible to compensate for the rotations by attaching the ends of the arms to the anchoring points via actuators driven by piezoelectric motors. The actuators would be programmed to null the rotation by varying the effective length of the arms in synchronism with the vertical motion. Potentially there might be a problem with high-frequency vibration generated by these actuators, but since the force they create is directed along the arms in a direction approximately perpendicular to the axial translation, the effect of this vibration would be small and relatively easy to eliminate by suitable averaging. Deliberate geometrical changes in the alignment and rotational and horizontal velocities can be produced by these actuators to investigate the magnitude of errors caused by the various residual misalignments of the apparatus.

We assume specific dimensions in order to more easily discuss small effects which affect the measurement. The suspended object is a light and non-conducting thin-walled tube or framework about 750 mm long and with a radius, R, of 140 mm. It is restrained by six light but stiff tubular rods 140 mm long attached tangentially, three near each end in a plane perpendicular to the axis of the main cylinder, as shown in figure 1.

A linear movement of z mm from the origin, where the rods are coplanar, is accompanied by a rotation of the tube about its axis by an angle of $z^2/2R^2$ rad; it can be seen that the sense of the rotation is independent of the sign of the displacement. The coil will be displaced symmetrically about the origin and, for a displacement of ± 20 mm, the maximum angle is 0.01 radians, or about 0.6°, but over a ± 10 mm length which might be used to gather data, the angle is only 0.0025 rad, or about 0.16°. At the origin, the rotation and its time variation are zero. These rotations combined with the interaction with a slightly unsymmetrical coil and magnet flux are likely to produce a small torque and induced voltage, but according to the above analysis, the result is unaffected since the torque and the angular velocity both arise from the mechanical structure of the balance. Under these circumstances the operation of the watt balance will cause their effects on the measurement to cancel.

5. Mechanical alignment

This is the aspect which has caused most difficulty for orthodox watt balances, but which is considerably simplified in the present proposal. No significant displacement occurs for four of the degrees of freedom, namely x and y displacements and rotations about these axes. The rotation about the z-axis does not affect the result, as indicated above.

The interferometer beam must be aligned with the vertical to ensure that the velocity of the frame is measured correctly along the vertical as $u_{z'}$. As is commonly done in gravimeter alignment, the interferometer axis can be aligned to the vertical using reflection from a liquid surface.

According to equation (9), no other alignment is necessary, but since the coil mounting is not infinitely stiff, reduction of any alignment changes between the weighing and moving phases of the measurement is still highly desirable. The motion of the coil can be aligned with the vertical interferometer beam by seeking no walk-off of the beam returning from the moving cube-corner retroreflector as the tube is translated along the z-axis, using a quadrant photocell to detect x or y offsets. A sensitivity of one micrometre in a total travel distance of 50 mm should be achievable. This corresponds to an angle of 2×10^{-5} rad. Alignment of the magnet with respect to the already-established vertical direction of the coil motion can be accomplished by making small translational and angular rotational offsets of the main magnet with respect to both the x and y axes by seeking equal increases in I about the minimum value for a given value of M and then setting to the mean offset position. These alignments are much less critical than for a conventional watt balance. Alternatively, if the suspension rods were fitted with sensitive strain gauges, the apparatus could be assembled with close to zero strain in the rods and changes in strain with changes of current in the coil could be used to align the magnet and coil.

6. Advantages of this proposal

(1) The performance of this ‘balance’ in the weighing mode should at least equal that of a conventional knife-edge balance. Strip-hinge hysteresis is known to be less than knife-edge hysteresis.

(2) The rigidity with regard to the five unwanted degrees of freedom implies that the difficult alignments of the unrestrained coil of a conventional watt balance are much less critical. This reduces the number of systematic errors that have to be evaluated and makes the proposal attractive for ‘routine’ future re-establishment of the kilogram unit. As a further example, if M were to be slightly mis-centred so that its weight acts along a vertical which is parallel to the central axis, the rigidity of the suspension ensures that weighing accuracy is nevertheless not compromised. This property is possessed by mass-comparison weighing instruments of a similar design.

(3) It is possible to incorporate a freely falling object into the apparatus with the advantage that the value of g can thereby be obtained when and where it is needed, i.e., adjacent to M and as part of a measurement run.

(4) The balance is compact, relatively robust and fairly simple to construct and use. This should reduce the costs involved in setting up and operating it.

In all, the performance of this proposed apparatus should at least equal that of the best of the other watt balance designs but would be easier to set up and operate.

7. A watt balance based on torque and angular velocity

Another possible new design of watt balance takes advantage of the known excellent weighing characteristics of the beam balance but simplifies the construction of the apparatus by separating the mechanical and electrical parts of the measurement. Figure 3 shows a diagram of this apparatus. The main mass pan incorporating a retroreflector for the interferometric measurement of its vertical velocity is suspended from one arm of the balance. A coil is attached to the beam of the balance and is placed in a radial magnetic field so that when a current is passed through it a torque is produced about an axis along the main knife-edge of the balance. A second, similar coil, the drive coil, is attached to the other side of the balance to provide a small torque to rotate the balance beam in moving mode. The weighing process is identical to that of a conventional watt balance: a mass is added or removed from the main mass pan and a measurement is made of the difference in the currents in the coil required to maintain equilibrium in the two states of the mass. As described above the balance is offset, in this case by the addition of a tare mass T to the other arm of the balance, to produce a current change that is symmetrical about zero. In the moving phase the interferometer measures the virtual vertical velocity of the mass M by measuring the velocity of the mass pan support when it is driven at constant speed by a small current in the drive coil. The voltage from the main coil is measured during this phase. The results are combined in the usual way to relate electrical and mechanical virtual power. This configuration of watt balance equates the torque due to the coil to the torque due to the weight of the mass acting over the (same) length of the balance arm as the term $\partial\theta_y/\partial z'$ in equation (6) is arranged to be the only such term of appreciable size. The angular velocity of the coil is determined by the measured velocity of the mass pan and the length of the balance arm. The length of the balance arm is eliminated between the two measurements, assuming that the length is the same in both measurements. This assumption can be tested by investigating the consistency between successive

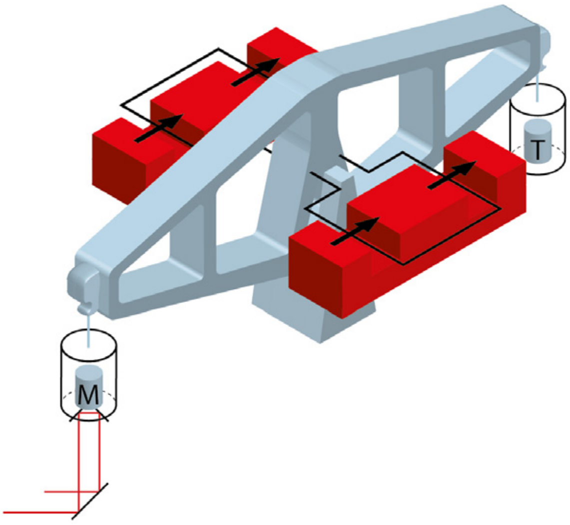

Figure 3. A view of a possible watt balance based on generating torque about the axis of the main knife edge of a balance using coils attached to the beam in conjunction with fixed magnets. In moving mode the vertical velocity of the mass pan is measured with a laser interferometer.

weighing or moving measurements in an interleaved set of such measurements. However if the arm length changes slowly and smoothly between successive measurements such changes can be eliminated readily from the results of the measurement. A possible alternative would be to adopt the room temperature variant of the simultaneous weighing and moving technique for the watt balance [8] which, with care, would eliminate the effect of such changes. In this case the main coil would be wound with a bifilar winding which provides a relatively simple method of separating the induced voltage and the weighing current which are present simultaneously. Adoption of this technique would also eliminate the heating effects of the current on the dimensions of the coil and the field of the magnet. However, if conventional watt balance operation was desired, the uncertainty contributions arising from the heating effect of the weighing current could be reduced to negligible levels by: careful design of the magnet to reduce its temperature coefficient, control of the magnetic field to reduce its value close to zero at the ends of the coil and the employment of a conventional bifilar heater winding.

The alignment requirements are identical to those of the previous design. If the coil mounting was carefully designed and fitted with sensitive strain gauges these could be used to align the magnet and coil to minimize unwanted torques and forces. However, because the extremely stiff mounting and the rigidity of the central knife of the balance should provide a very high degree of rejection of such torques and forces, a simple check that the result is independent of the position and angle of the magnet, combined with precise construction, should suffice for a given level of uncertainty.

8. Advantages of this proposal

1. It uses a conventional knife-edge balance which has been proven to achieve the required performance when operated as a conventional watt balance.
2. The magnet can be very well shielded and can be remote from the mass. This will reduce unwanted magnetic fields in the vicinity of the mass, allowing the use of stainless steel and platinum–iridium masses without significant corrections for their susceptibility.
3. The extremely stiff mounting of the coil and the rigidity of the balance knife minimize changes in alignment, and therefore changes to the terms in equation (6), between moving and weighing modes.

As this balance is based on improvements to technology used in existing watt balances such as that described in [5], which is capable of an uncertainty close to or better than 2 parts in 10^8, the performance of this proposed, simpler, apparatus should be at least equal to this.

9. Adaptation to larger and smaller masses

The principles and the two designs of watt balance described above simplify the adaptation of the watt balance both to larger and to smaller masses. For smaller masses both designs of balance could be produced using micro-electro-mechanical-systems (MEMS) techniques with the possibility of changing the actuation mechanism from electromagnetic to electrostatic. This would allow the construction of systems in the gram to milligram mass range. For larger masses, or for the generation of large forces, more powerful electromagnetic actuators would be needed but may allow novel applications in situations where it is undesirable to transport masses and employ conventional mass calibration techniques. The elimination of the need to carry out difficult alignment procedures would simplify the construction of such devices. However there would still be a need to verify the accuracy of the completed measurement systems. This could be achieved initially by building both types of balance and comparing the results. The error mechanisms will be different in the two balances and agreement to within a predicted uncertainty would give considerable confidence in the construction and operation of both devices. Almost all the equipment required to operate the balances would be common to both and therefore the added cost of this approach would reside in the fabrication costs of one of the balances, which should be a small proportion of the cost of setting up such a facility.

10. A horizontal watt balance for relating Planck's constant to inertial mass

All watt balances so far, either working or in an advanced evaluation stage, or as in this proposal, relate the force exerted by a mass in the Earth's gravitational field, g, to that of a current-carrying coil in a magnetic flux. The value of g is measured with a relative accuracy of parts in 10^9 by interferometric observation of the acceleration of a freely falling object.

Alternatively, our proposed suspension could be applied, following a suggestion by Cabiati [9, 10], to measure the acceleration a of an object of mass M suspended so as to move freely *horizontally* when subjected to a force generated by an electrical current I enclosing a magnetic flux Φ. Φ is eliminated as usual by a second measurement of the voltage induced in the coil when it moves with velocity v. Thus

$$Ma + F_1 + F_2 + F_3 = IV/v, \tag{10}$$

where F_1 is the elastic restoring force of the suspension, F_2 is the sum of viscous forces proportional to v and F_3 is the inertial force of the total suspended mass, which is proportional to a. In our proposed design $F_1 + F_2$ is very small compared to Ma, and it and F_3 can be eliminated by subtracting the results of current measurements made with M present, and then removed, whilst keeping a and v constant. At the start of the acceleration F_3 will be temporarily augmented by elastic forces stored in the tube between the coil and M and sufficient time must be allowed after the commencement of the accelerating mode for these forces to be damped out. I must be generated by a true current source, i.e., one of very high differential output impedance of the order of 10^{12} Ω to avoid generating a damping contribution to F_2.

The well-advanced searches for systematic (type B) errors pertinent to free-fall absolute gravimeters and orthodox watt balances also apply to a horizontal apparatus, and this should greatly reduce the time taken for their evaluation. There are also several advantages.

(1) Since a can be chosen to have any value which yields sufficient measurement time and does not compromise the resolution of the measurement, there is greater freedom to choose and vary the values of M and I to aid the search for type B errors. In particular, I could be about 0.1 mA, which would produce about 1 V across a quantum Hall device used directly with the apparatus. The considerable advantage would be a 'resistor' having no ageing, temperature or pressure coefficients and whose value does not change on transport.

(2) There is no need for a separate and expensive apparatus to measure g and carry out the difficult and time-consuming task of relating its value at the point in space where it is measured to its value at the mass centre of M.

(3) The stiffness of our proposed suspension to all forces and torques, other than the torque about a horizontal axis, makes alignment easier.

(4) The simplicity of the mechanical design and its comparative immunity to vibration make it more suitable for periodic re-evaluation of a primary mass artefact.

Whilst there are advantages to this approach there are a number of sources of uncertainty which do not arise in a conventional watt balance and which may limit the ultimate uncertainty which can be achieved by the technique. This is because some of the effects of the accelerations required in the mass-determining part of the measurement have no counterpart in the moving part of the experiment. For example: the rotation associated with horizontal motion will couple the moment of inertia of the frame into the effective mass of the system. This effect will be increased if the axis of rotation does not pass through the centre of mass of the frame, which might change when the test mass M is in place. The vertical stiffness of the suspension is critical, especially as frame displacements will involve the frame mass. When the test mass is in place its moment of inertia will also affect the force required to accelerate the frame and a further force will be required if the centre of mass of the test mass is not on the axis of rotation.

In addition F_1, the elastic restoring force of the strip hinges which, in principle, is zero at the important symmetrical point, will depend slightly on the load they have to support, that is, on whether the mass is in place or removed. At the extreme ±10 mm points of the observed region, the restoring force from properly designed strip hinges is likely to be of the order of 10^{-4} to 10^{-5} N, and is thus a very significant force whose change under the increased load when the mass is in position must be accounted for.

All of these effects would require careful investigation and appropriate correction.

11. Design of a horizontal apparatus

There are some aspects of the design of a horizontal inertial apparatus which are different from those of a vertical apparatus. The diagram of figure 1 still applies with rods 3 and 6 in the same horizontal plane and the rest of the object and suspension rods above them. Rods 1 and 4 together are in tension, bearing a total force of $M'g/\sqrt{3}$ where M' is the total suspended mass. Rods 2 and 5 bear the same force, but in compression. It is easy to arrange that their strip hinges are nevertheless also in tension. There is very little force exerted by rods 3 and 6. The coil is wound on one end of the tube and is surrounded by a magnet giving a radial magnetic flux centred on a horizontal axis. The mass to be calibrated is placed centrally on and off a rest connected to the tube by a mechanism not shown in detail.

Since the force exerted by the coil is with reference to the magnet, the displacement of the mass must also be referenced to the magnet. Hence the moving interferometer reflector is rigidly connected to the tube and positioned on its axis and the reference reflector and beam-divider are rigidly attached to the magnet. Built-in vibration isolation can be attained by also suspending the magnet by six jointed rods, in the same manner as the tube but with motion damping added. The situation is the same as the employment of a 'superspring' mounting for the fixed cube-corner in a free-fall gravimeter [6]. Doing this should considerably reduce the number of observations needed to attain a given resolution, but the mass of the magnet, which accelerates away under the influence of the recoil force, must be measured relative to the mass to be calibrated with an accuracy of the order of 1 in 10^5.

A possibility for avoiding the recoil, and therefore the need to know the mass of the magnet accurately, is to arrange a complete duplicate coil and mass-mounting assembly in the reversed direction on the other side of the magnet, in a similar radial flux. If both coils are operated simultaneously and symmetrically in opposition with the same current, the two assemblies will move apart with equal and opposite accelerations and the total recoil force on the composite magnet will be very small. The distance measured is the total distance between the cube-corner mounted on one coil assembly and that mounted on the other. Each coil assembly would need to be mounted in its own sub-frame so that they can individually be aligned along the direction of maximum induced force.

The coil can be aligned to move along a horizontal line by seeking zero acceleration when $I = 0$. This is not critical as any remnant acceleration will be eliminated by current and acceleration reversal. Alignment of the flux of the magnet with respect to this line can be accomplished by the methods mentioned previously.

For this design a practically achievable acceleration is likely to be less than 0.01 g and this makes removal of the effects of residual forces and accelerations more difficult. While we feel that the horizontal design represents an interesting route to realising the mass unit, the two other, more conventional, watt balance designs described in this paper would probably require less work, fewer corrections and yield a lower ultimate uncertainty.

12. Conclusion

We have demonstrated that a watt balance having its coil motion constrained so that its configuration does not change appreciably between the weighing and moving modes is insensitive to misalignments involving the balance and the electromagnetic interactions between the coil and magnet, providing that the motion of the mass pan is measured along the gravitational vertical. Apparatus of this kind could well be more appropriate for the next generation of watt balances which will be aimed at periodic regeneration of the kilogram from its proposed definition in terms of a fixed value of the Planck constant. The techniques described would also facilitate MEMS watt balance techniques for the direct realization of the mass scale, from its proposed definition, at gram and milligram levels and also, by scaling up the balances, direct realizations at levels above 1 kg could be achieved.

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